THERMODIFFUSIOPHORETIC DEPOSITION OF HIGHLY DISPERSED PARTICLES ON THE SURFACE OF A CYLINDER

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A theoretical study is made of the deposition of particles on the surface of a cylinder. It is shown that the flow of particles being deposited is proportional to the flows of heat and substance passing over the cylinder surface.

Several studies have examined the thermodiffusiophoretic deposition of highly dispersed particles in plane and cylindrical channels [1-4]. They have shown that the thermophoretic and diffusiophoretic forces can exert a significant effect on the deposition of particles from gas flows which are nonuniform with respect to temperature and concentration.

When a gas flows about a transversely positioned cylinder being cooled, thermodiffusiophoretic deposition of highly dispersed particles should also occur on the outside surface of the cylinder. However, there has not yet been a sufficiently complete theoretical description of this process.

In connection with this, the present work theoretically describes the deposition of highly dispersed particles from a laminar flow of a binary gas mixture on the surface of an infinite cylinder undergoing cooling. Here, we assume that the cylinder surface can absorb molecules of both the first and second components of the gas mixture. No reverse flows form near the cylinder. The dimensions of the particles are much smaller than the cylinder radius.

We will examine the steady transport of particles to the cylinder surface. Here, the values of particle concentration Φ, and the temperature T and relative concentrations C_1 and C_2 of the molecules of the gas phase in the flow incident on the surface will be assumed to be known. No limitations are imposed on the gradients of C_1 and T. The values of C_1, C_2, and T at the surface do not depend on the coordinate of the z axis (generatrices of the cylinder parallel to the z axis).

In the vicinity of the cylinder, the densities of the flows of molecules of the first and second components j_1 and j_2, heat j_t, and the particles j_p satisfy the conservation equations

$$\operatorname{div}\mathbf{j}_1 = 0,\ \operatorname{div}\mathbf{j}_2 = 0,\ \operatorname{div}\mathbf{j}_t = 0,\ \operatorname{div}\mathbf{j}_p = 0, \tag{1}$$

$$\mathbf{j}_1 = n_1\mathbf{U} - \frac{n^2 m_2}{\rho} D_{12}\left(\nabla C_1 + k_t \frac{\nabla T}{T}\right), \tag{2}$$

$$\mathbf{j}_2 = n_2\mathbf{U} - \frac{n^2 m_2}{\rho} D_{12}\left(\nabla C_2 - k_t \frac{\nabla T}{T}\right), \tag{3}$$

$$\mathbf{j}_t = (\alpha_1\mathbf{j}_1 + \alpha_2\mathbf{j}_2)\cdot kT - \varkappa\nabla T, \tag{4}$$

$$\mathbf{j}_p = (\mathbf{U} + \mathbf{U}_{td})\cdot\Phi. \tag{5}$$

The rate of thermodiffusiophoresis in (5) can be represented as follows [5-7]:

$$\mathbf{U}_{td} = f_c D_{12}\nabla C_1 - f_t \nu \frac{\nabla T}{T}, \tag{6}$$

where f_c and f_t are coefficients dependent on the Knudsen number.

It can be shown on the basis of Eq. (1) that identical [8] flows of molecules of the first and second kinds (Q_1 and Q_2), heat Q_t, and aerosol particles Q_p pass over any closed

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surface S enveloping the cylinder (the surface being bounded by two parallel planes located the same distance b_c perpendicular to generatrices of the cylinder):

$$Q_1 = \int_S \mathbf{j}_1 \cdot d\mathbf{S} = \text{const},\ Q_2 = \int_S \mathbf{j}_2 \cdot d\mathbf{S} = \text{const},$$
$$Q_t = \int_S \mathbf{j}_t \cdot d\mathbf{S} = \text{const},\ Q_p = \int_S \mathbf{j}_p \cdot d\mathbf{S} = \text{const}. \tag{7}$$

In (7), dS is a vector element of the surface, the direction of which will be assumed to be opposite to the direction of an external normal to the surface. With such a choice of the direction for dS, the flows of molecules, heat, and particles passing over the outside surface of the cylinder will correspond to positive values of Q_1, Q_2, Q_t, and Q_p.

If S approaches infinity with a fixed length b_c, then U, n_1, n_2, and Φ will approach the unperturbed values they have in the presence of the cylinder U_∞, $n_{1\infty}$, $n_{2\infty}$, T_∞, Φ_∞, i.e.,

$$\mathbf{U} = \mathbf{U}_\infty + \delta\mathbf{U},\ n_1 = n_{1\infty} + \delta n_1,\ n_2 = n_{2\infty} + \delta n_2,$$
$$C_1 = C_{1\infty} + \delta C_1,\ C_2 = C_{2\infty} + \delta C_2,\ T = T_\infty + \delta T, \tag{8}$$
$$\Phi = \Phi_\infty + \delta\Phi.$$

In (8), δ denotes small corrections to the main quantities. Inserting (8) into (7) and considering the smallness of the corrections in (8), we obtain

$$Q_1 = \int_S \left[(n_{1\infty}\delta\mathbf{U} + \mathbf{U}_\infty \delta n_1) - \frac{n_\infty^2 m_2}{\rho_\infty} D_{12\infty} \left(\nabla\delta C_1 + \frac{k_{t\infty}}{T_\infty} \nabla\delta T \right) \right] d\mathbf{S},$$

$$Q_2 = \int_S \left[(n_{2\infty}\delta\mathbf{U} + \mathbf{U}_\infty \delta n_2) - \frac{n_\infty^2 m_2}{\rho_\infty} D_{12\infty} \left(\nabla\delta C_2 + \frac{k_{t\infty}}{T_\infty} \nabla\delta T \right) \right] d\mathbf{S},$$

$$Q_T = (\alpha_{1\infty} Q_1 + \alpha_{2\infty} Q_2) kT_\infty + \int_S \{ \mathbf{U}_\infty k [n_{1\infty}\delta(\alpha_1 T) + n_{2\infty}\delta(\alpha_2 T)] - \varkappa_\infty \nabla\delta T \} d\mathbf{S}, \tag{9}$$

$$Q_p = \int_S \left[\delta\mathbf{U} - f_{T\infty} \frac{\nu_\infty \nabla\delta T}{T_\infty} - f_{c\infty} D_{12\infty} \nabla C_1 \right) \Phi_\infty + \mathbf{U}_\infty \delta\Phi \right] d\mathbf{S}.$$

At large distances from the cylinder, conservation equations (1) are reduced to the following form:

$$\operatorname{div} \delta\mathbf{U} = -\mathbf{U}_\infty \frac{\nabla\delta\rho}{\rho_\infty}, \tag{10}$$

$$n_{1\infty} \operatorname{div} \delta\mathbf{U} + \mathbf{U}_\infty \nabla\delta n_1 = D_{12\infty} \frac{n_\infty^2 m_2}{\rho_\infty} \left[\Delta\delta C_1 + k_{t\infty} \frac{\Delta\delta T}{T_\infty} \right], \tag{11}$$

$$k\mathbf{U}_\infty (\nabla\delta(\alpha_1 T) n_{1\infty} + n_{2\infty} \nabla\delta(\alpha_2 T)] = \varkappa \Delta\delta T, \tag{12}$$

$$\Phi_\infty \left[\operatorname{div} \delta\mathbf{U} - f_{t\infty} \frac{\nu_\infty}{T_\infty} \Delta\delta T - f_{c\infty} D_{12\infty} \Delta\delta C_1 \right] + \mathbf{U}_\infty \nabla\delta\Phi = 0, \tag{13}$$

where Δ is the Laplace operator. Having expressed div δU, $\Delta\delta C_1$ and $\Delta\delta T$ in (13) through U_∞ by means of (10)-(12) and having then integrated (13), we obtain

$$\Phi = \Phi_\infty \left\{ 1 + \frac{\delta\rho}{\rho_\infty} + \frac{f_{c\infty}\rho_\infty}{n_\infty^2 m_2} \left[\delta n_1 - n_{1\infty} \frac{\delta\rho}{\rho_\infty} \right] + [f_{t\infty}\nu_\infty - f_{c\infty} k_{t\infty} D_{12\infty}] \frac{k}{\varkappa_\infty T_\infty} [n_{1\infty}\delta(\alpha_1 T) + n_{2\infty}\delta(\alpha_2 T)] \right\}. \tag{14}$$

Having inserted (14) into the expression for Q_p (9) and then comparing the expressions entering into (9), we obtain the following formula for Q_p

$$Q_p = \Phi_\infty \left\{ \frac{1}{\rho_\infty} [m_1 Q_1 + m_2 Q_2] + f_{c\infty} \left[\frac{n_{2\infty}}{n_\infty^2} Q_1 - \frac{n_{1\infty}}{n_\infty^2} Q_2 \right] + \frac{1}{\varkappa_\infty T_\infty} [f_{t\infty}\nu_\infty - f_{t\infty} k_{T\infty} D_{12\infty}] [Q_t - kT_\infty (\alpha_{1\infty} Q_1 + \alpha_{2\infty} Q_2)] \right\}.$$

It follows from (15) that, knowing the flows of heat Q_t and of molecules of the first and second types Q_1 and Q_2 passing over a cylinder surface of length b_c, we can find the number of particles Q_p settling on this surface per unit of time. Particle deposition occurs when the first part of (15) is positive. Here, the flows Q_1 and Q_2 in (15) can be found either theoretically or experimentally. It is apparent from (15) that when no molecules are absorbed by the surface ($Q_1 = Q_2 = 0$), particles are deposited under the influence of the thermophoretic force only during cooling of the gas flow, i.e., when $Q_t > 0$.

The following formula can be used to find a coefficient E [9] characterizing the deposition of aerosol particles on a circular cylinder as a result of nonuniform distribution of T, C_1, and C_2. The coefficient is equal to the ratio of the maximum impact parameter to the cylinder radius R. Here, Q_p is known

$$E = \frac{Q_p}{2RU_\infty b_c \Phi_\infty}. \tag{16}$$

The same coefficient for deposition due to Brownian diffusion can be evaluated from the following formula for Reynolds numbers less than unity

$$E^* = 2{,}9\,\mathrm{Pe}^{-2/3}(2 - \ln \mathrm{Re})^{-1/3}, \tag{17}$$

where $\mathrm{Pe} = 2U_\infty R/D_B$; $\mathrm{Re} = 2U_\infty R/\nu$; D_B is the coefficient of Brownian diffusion; ν is the kinematic viscosity. Estimates that were made showed that the flow (15) may be considerably greater than the flow of particles due to Brownian diffusion. In fact, the value of $A = E/E^*$ in nitrogen at a cylinder surface temperature T = 300°K, pressure $p = 10^5$ Pa, Re = 0.6, and $\Delta T = T_\infty - T_S = 40$°K is equal to 66 for particles with a radius $a = 4 \cdot 10^{-7}$ m and A = 15 at $a = 4 \cdot 10^{-8}$ m.

In a binary gas mixture consisting of molecules of water and nitrogen vapors, with $T_\infty = T_S = 300$°K, $p = 10^5$ Pa, Re = 0.6, a relative concentration of water vapor at the cylinder surface $C_{1S} = 0.015$, and $\Delta C_1 = C_{1\infty} - C_{1S} = 0.2$, the value of A = 220 for particles with $a = 4 \cdot 10^{-7}$ m and $A \approx 19$ for particles with $a = 4 \cdot 10^{-8}$ m.

NOTATION

j_1, j_2, densities of flows of molecules of the first and second types; j_t, j_p, densities of flows of heat and particles; U, mass velocity of gas; $\rho = m_1 n_1 + m_2 n_2$; $C_1 = n_1/n$; $C_2 = n_2/n$; $n = n_1 + n_2$; m_1, m_2, n_1, n_2, masses and concentrations of the molecules of the first and second components; α_1, α_2, isobaric specific heats for molecules with m_1 and m_2 [5]; $\varkappa$, thermal conductivity of the gas mixture; k, Boltzmann constant; D_{12}, diffusion coefficient; k_t, coefficient of thermodiffusion ratio; ν, kinematic viscosity; Φ, particle concentration; Q_1, Q_2, Q_t, and Φ, flows of molecules of the first and second types, heat, and particles.

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